

Simple blog engine with shape functors and generic eliminators for ADTs

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An experiment on a blog engine is complicated enough, to be a real world like problem. It is small enough to code it in a few hours after work.

This will be confusing as the same phenomenon has many names in the literature.

- Generic eliminator
- Eliminator
- Catamorphism
- Initial algebra
- Template function

The full power of the generic eliminators are connected to dependent typed programming.

Haskell is not dependent yet, let's use a simple approach.

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This presentation is about the definitions and the practical use of generic eliminators.

```
-- Abstract deepsense. (Matthias Fishmann)
module Eliminator Theory where
```

Basic example

```
data List a
  = Empty
  | Cons a (List a)

length :: List a -> Int
length Empty      = 0
length (Cons _ xs) = 1 + length xs
```

Abstract the recursion

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```
list_elim :: (b, a -> b -> b) -> List a -> b
list_elim (empty, cons) Empty          = empty
list_elim (empty, cons) (Cons a as) =
  cons a (list_elim (empty, cons) as)
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```

```
length_alg :: (Int, a -> Int -> Int)
length_alg = (0, \_ n -> 1 + n)
```

```
length' :: List a -> Int
length' = list_elim length_alg
```

Abstract the list shape

Shape functors.

Eliminators for an ADT can be separated into the shape of the ADT and the recursion scheme.

```
data ListShape a rec
  = EmptyS
  | ConsS a rec
  deriving (Show)
```

```
newtype Fix f = In { unFix :: f (Fix f) }
```

```
type ListF a = Fix (ListShape a)
```

```
e   = In EmptyS
ae  = In (ConsS 1 e)
aae = In (ConsS 2 ae)
```

Abstract the list shape

```
listElim :: (b, a -> b -> b) -> ListF a -> b
listElim (empty, cons) (In EmptyS)      = empty
listElim (empty, cons) (In (ConsS a s)) =
    cons a (listElim (empty, cons) s)

length'' = listElim (0, (\_ n -> (1 + n)))
```

Abstract the cases of list shape

Let's rename `listElim` to `listCata` as we abstracting away from the value processing.

```
listCata :: (ListShape a s -> s) -> ListF a -> s
listCata alg (In EmptyS)      = alg EmptyS
listCata alg (In (ConsS a s)) = alg (ConsS a (listCata alg s))
```

```
lengthAlg :: ListShape a Int -> Int
lengthAlg EmptyS      = 0
lengthAlg (ConsS _ n) = 1 + n
```

```
length''' = listCata lengthAlg
```

Shape functor

```
instance Functor (ListShape a) where
  fmap f EmptyS      = EmptyS
  fmap f (ConsS a s) = ConsS a (f s)
```

Catamorphism

Let's abstract the shape functor. In Category theory the algebras are defined for functors. Many algebra can be defined for the given functor.

```
-- newtype Fix f = In { unFix :: f (Fix f) }

type Algebra f a = f a -> a

cata :: Functor f => Algebra f s -> Fix f -> s
cata alg = alg          -- Compute the result from the partials
    . fmap (cata alg) -- Compute the partial results
    . unFix            -- Step inside

length'''' = cata lengthAlg
```

Catamorphism is a recursion scheme.

Factorial?

Catamorphisms are not powerful enough, there is a zoo of morphisms. We need another type of morphism to be able to define the factorial function.

<http://hackage.haskell.org/package/fixplate>

```
-- Concrete nonsense.  
module Elimimators.Practice where
```

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Use Fix, Shape functors and cata if your data types are tend to be recursive and there is a high probability of changes

Use Eliminator otherwise.

The godfather of all eliminators

First and well known lazy generic eliminator in every programming language!

```
boolElim t f e = if e then t else f
```

or

```
boolElim' t f e = case e of
  True  -> t
  False -> f
```

More eliminators

With Haskell we can create eliminators for every ADT, based on the structure of the ADT. With laziness generic eliminators can serve as template functions for the values we work with.

```
maybeElim n j m = case m of
  Nothing -> n
  Just x   -> j x
```

```
eitherElim l r e = case e of
  Left x   -> l x
  Right y  -> r y
```

Composition of eliminators comes from the structural induction on the shape of ADT.

```
compExample =  
  eitherElim  
    (eitherElim  
      (show . (1+))  
      ("x=" ++))  
    (maybeElim "NaN" (show . floor))
```

Using intendation helps a lot. It is very similar to the pointfree style.

Design recipe

- Create an ADT

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- Encapsulate this definitions in a module
- Sometimes it is useful to add a typed hole, which can carry out information
- Create algebras to define functions with eliminators

In real world examples the information usually organized in a tree shaped data.

```
data Entry a = Entry {  
    e_hole  :: a  
    , e_lines :: Pandoc  
} deriving (Functor, Eq, Show)
```

```
type EntryAlgebra a b = (a -> Pandoc -> b)
```

```
entryElim :: EntryAlgebra a b -> Entry a -> b  
entryElim alg (Entry hole lines) = alg hole lines
```

Type hole in Entry. With a type hole we can express more computational power and can convert our regular data type to a shape functor, and if we need we can use it in Fix computations.


```
data TopicName a = TopicName {  
    tn_hole :: a  
    , tn_name :: Pandoc  
} deriving (Functor, Eq, Show)
```

```
type TopicNameAlgebra a b = (a -> Pandoc -> b)
```

```
topicNameElim :: TopicNameAlgebra a b -> TopicName a -> b  
topicNameElim alg (TopicName hole name) = alg hole name
```

Topic

```
data Topic a = Topic {  
    t_hole      :: a  
    , t_topicName :: TopicName a  
    , t_entries  :: [Entry a]  
} deriving (Functor, Eq, Show)
```

```
data Topic a = Topic {
  t_hole      :: a
  , t_topicName :: TopicName a
  , t_entries  :: [Entry a]
} deriving (Functor, Eq, Show)
```

How to define an eliminator and algebras for Topic?

```
type TopicAlgebra a t e es p =
  ( TopicNameAlgebra a t
  , EntryAlgebra a e
  , ListAlgebra e es
  , a -> t -> es -> p)

topicElim :: TopicAlgebra a t e es p -> Topic a -> p
topicElim (topicNameAlg, entryAlg, entriesAlg, combine)
  (Topic hole topicName entries)
= combine
  hole
  (topicNameElim topicNameAlg topicName)
  (listElim_ entriesAlg (entryElim entryAlg <$> entries))
```

```
data Blog a = Blog {
    b_hole      :: a
  , b_summary  :: Pandoc
  , b_topics   :: [Topic a]
} deriving (Functor, Eq, Show)

type BlogAlgebra a t e es p bs b =
    ( TopicAlgebra a t e es p
    , ListAlgebra p bs
    , a -> Pandoc -> bs -> b )

blogElim :: BlogAlgebra a t e es p bs b -> Blog a -> b
blogElim (topic, topicList, combine)
    (Blog hole summary topics)
= combine
    hole
    summary
    (listElim_ topicList (topicElim topic <$> topics))
```

```
renderPages :: FilePath -> (NavPath -> Html -> Html)
              -> Blog FileProperties -> IO ()
renderPages outDir frame = blogElim render where
  render = (topic, sequence_, topicList)
  topic  = (topicName, entry, entryList, topicNameEntryList)

  entry fp pandoc = do
    writeFile (outDir </> (markdownPathToHTMLPathFP fp))
              (renderHtml . frame NavBackward $ pandoc2html pandoc)
    return (fp, firstHeader pandoc)

  entryList = (return [], \x xs -> (:) <$> x <*> xs)
  topicList _ _ ts = ts
  sequence_ = (return (), (>>)) -- Monoid instance of monads
```

```
topicName fp pandoc = do
  createDirectoryIfMissing True $ outDir </> markdownPathToHTMLDir fp
  return (\content -> writeFile
          (outDir </> (markdownPathToHTMLPathFP fp))
          (renderHtml $ frame NavInPlace content)
        , pandoc
        )
```

```
topicNameEntryList _ topicName entryList = do
  (topicPage, pandoc) <- topicName
  headers <- entryList
  topicPage $ do -- :: Html
    pandoc2panel pandoc
    topicsList headers
```

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Solutions:

- Create an ADT for the algebra and name the constructors
- Use the new Symbol types as type parameter to name the different cases

Named parameters

Use named parameters

```
data Param (n :: Symbol) a = Param a
```

```
maybeElimNamed
```

```
  :: (Param "nothing" b) -> (Param "just" (a -> b)) -> Maybe a -> b
maybeElimNamed (Param nothing) (Param just) = \case
  Nothing -> nothing
  Just x   -> just x
```

```
test = maybeElimNamed
```

```
  (Param 0      :: Param "nothing" Int)
  (Param (1+)  :: Param "just" (Int -> Int))
```

Connection to lenses

- Lenses are coalgebras, composition works via function composition
- Eliminators use algebras, composition works via tupling
- Eliminators are like universal properties for an ADT

More...

- Use template haskell to generate eliminators from the ADT
- Use generics-sop library to generate eliminators
- Create a library

Conclusion

- Similar to point free style
- Algorithms are compact, but still understandable
- Composition done by chaining or tupling of algebras

<https://github.com/andorp/andorp.github.io/tree/master/haskell>